



BOARD QUESTION PAPER : MARCH 2019

MATHEMATICS AND STATISTICS

Note:

- (1) All questions are compulsory.
- (2) Figures to the right indicate full marks.
- (3) The Question paper consists of **30** questions divided into **FOUR** sections **A, B, C, D**.
 - **Section A** contains **6** questions of **1 mark** each.
 - **Section B** contains **8** questions of **2 marks** each. (One of them has internal option)
 - **Section C** contains **6** questions of **3 marks** each. (Two of them have internal options)
 - **Section D** contains **10** questions of **4 marks** each. (Three of them have internal options)
- (4) For each **MCQ**, correct answer must be written along with its **alphabet**,
e.g., (A) / (B) / (C) / (D) etc.
In case of **MCQs**, (Q. No. **1** to **6**) evaluation would be done for the first attempt only.
- (5) Use of logarithmic table is allowed. Use of calculator is **not** allowed.
- (6) In **L.P.P.** only rough sketch of graph is expected. Graph paper is **not** necessary.
- (7) Start each section on new page only.

SECTION A

Select and write the most appropriate answer from the given alternative for each question:

1. The principal solutions of $\cot x = -\sqrt{3}$ are _____. [1]
(A) $\frac{\pi}{6}, \frac{5\pi}{6}$ (B) $\frac{5\pi}{6}, \frac{7\pi}{6}$
(C) $\frac{5\pi}{6}, \frac{11\pi}{6}$ (D) $\frac{\pi}{6}, \frac{11\pi}{6}$
2. The acute angle between the two planes $x + y + 2z = 3$ and $3x - 2y + 2z = 7$ is _____. [1]
(A) $\sin^{-1}\left(\frac{5}{\sqrt{102}}\right)$ (B) $\cos^{-1}\left(\frac{5}{\sqrt{102}}\right)$
(C) $\sin^{-1}\left(\frac{15}{\sqrt{102}}\right)$ (D) $\cos^{-1}\left(\frac{15}{\sqrt{102}}\right)$
3. The direction ratios of the line which is perpendicular to the lines with direction ratios $-1, 2, 2$ and $0, 2, 1$ are _____. [1]
(A) $-2, -1, -2$ (B) $2, 1, 2$
(C) $2, -1, -2$ (D) $-2, 1, -2$
4. If $f(x) = (1 + 2x)^{\frac{1}{x}}$, for $x \neq 0$ is continuous at $x = 0$, then $f(0) =$ _____. [1]
(A) e (B) e^2
(C) 0 (D) 2
5. $\int \frac{dx}{9x^2 + 1} =$ _____. [1]
(A) $\frac{1}{3} \tan^{-1}(2x) + c$ (B) $\frac{1}{3} \tan^{-1} x + c$
(C) $\frac{1}{3} \tan^{-1}(3x) + c$ (D) $\frac{1}{3} \tan^{-1}(6x) + c$



6. If $y = ae^{5x} + be^{-5x}$, then the differential equation is _____. [1]

(A) $\frac{d^2y}{dx^2} = 25y$ (B) $\frac{d^2y}{dx^2} = -25y$

(C) $\frac{d^2y}{dx^2} = -5y$ (D) $\frac{d^2y}{dx^2} = 5y$

SECTION B

7. Write the truth values of the following statements: [2]

i. 2 is a rational number and $\sqrt{2}$ is an irrational number.

ii. $2 + 3 = 5$ or $\sqrt{2} + \sqrt{3} = \sqrt{5}$

8. Find the volume of the parallelepiped, if the coterminus edges are given by the vectors $2\hat{i} + 5\hat{j} - 4\hat{k}$, $5\hat{i} + 7\hat{j} + 5\hat{k}$, $4\hat{i} + 5\hat{j} - 2\hat{k}$. [2]

OR

Find the value of p, if the vectors $\hat{i} - 2\hat{j} + \hat{k}$, $2\hat{i} - 5\hat{j} + p\hat{k}$ and $5\hat{i} - 9\hat{j} + 4\hat{k}$ are coplanar.

9. Show that the points A(-7, 4, -2), B(-2, 1, 0) and C (3, -2, 2) are collinear. [2]

10. Write the equation of the plane $3x + 4y - 2z = 5$ in the vector form [2]

11. If $y = x^x$, find $\frac{dy}{dx}$. [2]

12. Find the equation of tangent to the curve $y = x^2 + 4x + 1$ at (-1, -2). [2]

13. Evaluate: $\int \frac{e^x(1+x)}{\cos^2(xe^x)} dx$ [2]

14. Evaluate: $\int_0^{\frac{\pi}{2}} \sin^2 x dx$ [2]

SECTION C

15. In $\triangle ABC$, prove that

$$\sin\left(\frac{B-C}{2}\right) = \left(\frac{b-c}{a}\right) \cos\left(\frac{A}{2}\right) \quad [3]$$

OR

Show that $\sin^{-1}\left(\frac{5}{13}\right) + \cos^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{63}{16}\right)$

16. If A($\vec{a}$) and B ($\vec{b}$) are any two points in the space and R ($\vec{r}$) be a point on the line segment AB dividing it internally in the ratio m : n, then prove that $\vec{r} = \frac{m\vec{b} + n\vec{a}}{m+n}$ [3]

17. The equation of a line is $2x - 2 = 3y + 1 = 6z - 2$, find its direction ratios and also find the vector equation of the line. [3]



18. Discuss the continuity of the function

$$f(x) = \frac{\log(2+x) \log(2-x)}{\tan x}, \quad \text{for } x \neq 0$$

$$= 1 \quad \text{for } x = 0$$

at the point $x = 0$

[3]

19. The probability distribution of a random variable
- X
- , the number of defects per 10 meters of a fabric is given by

x	0	1	2	3	4
$P(X=x)$	0.45	0.35	0.15	0.03	0.02

Find the variance of X .

[3]

ORFor the following probability density function (p.d.f.) of X , find: (i) $P(X < 1)$, (ii) $P(|X| < 1)$

$$\text{if } f(x) = \frac{x^2}{18}, \quad -3 < x < 3$$

$$0, \quad \text{otherwise}$$

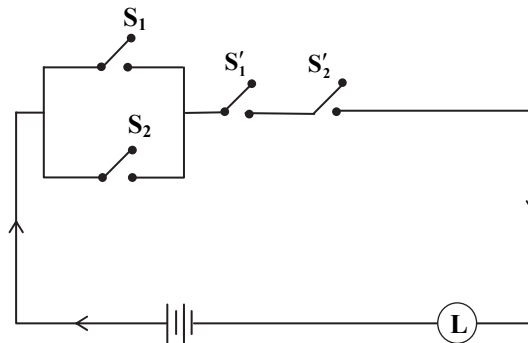
20. Given is
- $X \sim B(n, p)$
- .

If $E(X) = 6$, $\text{Var.}(X) = 4.2$, find n and p .

[3]

SECTION D

21. Find the symbolic form of the given switching circuit. Construct its switching table and interpret your result.



[4]

22. If three numbers are added, their sum is 2. If two times the second number is subtracted from the sum of first and third numbers we get 8 and if three times the first number is added to the sum of second and third numbers we get 4. Find the numbers using matrices.

[4]

23. In
- ΔABC
- , with usual notations prove that
- $b^2 = c^2 + a^2 - 2ca \cos B$

[4]

ORIn ΔABC , with usual notations prove that

$$(a-b)^2 \cos^2\left(\frac{C}{2}\right) + (a+b)^2 \sin^2\left(\frac{C}{2}\right) = c^2.$$

24. Find 'p' and 'q' if the equation
- $px^2 - 8xy + 3y^2 + 14x + 2y + q = 0$
- represents a pair of perpendicular lines.

[4]



25. Maximize: $z = 3x + 5y$ Subject to $3x + y \leq 21,$
 $x + 4y \leq 24,$ $x \geq 0, y \geq 0$ [4]
 $x + y \leq 9,$

26. If $x = f(t)$ and $y = g(t)$ are differentiable functions of t , then prove that y is a differentiable function of x and

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}, \text{ where } \frac{dx}{dt} \neq 0$$

- Hence find $\frac{dy}{dx}$ if $x = a \cos^2 t$ and $y = a \sin^2 t$. [4]

27. $f(x) = (x-1)(x-2)(x-3)$, $x \in [0, 4]$, find 'c' if LMVT can be applied. [4]

OR

A rod of 108 meters long is bent to form a rectangle. Find its dimensions if the area is maximum.

28. prove that : $\int \frac{dx}{\sqrt{x^2 + a^2}} = \log \left| x + \sqrt{x^2 + a^2} \right| + c$ [4]

29. Show that: $\int_0^{\frac{\pi}{4}} \log(1 + \tan x) dx = \frac{\pi}{8} \log 2$ [4]

30. Solve the differential equation:

$$\frac{dy}{dx} + y \sec x = \tan x$$
 [4]

OR

Solve the differential equation:

$$(x+y) \frac{dy}{dx} = 1$$